What is the Laplace Transform?

Discussion

- Operator (like the derivative)
- Notation/Letters

What is the Laplace Transform?

Laplace Transform

Definition 1. Let f(t) be a function on $[0, \infty)$. The **Laplace transform** of f is the function F defined by the integral

(1)
$$F(s) := \int_0^\infty e^{-st} f(t) dt.$$

The domain of F(s) is all the values of s for which the integral in (1) exists. The Laplace transform of f is denoted by both F and $\mathcal{L}\{f\}$.

More Discussion

- Fill in blank space
- Treat s like a constant...
- Domains

Calculating the Laplace Transform of Some Functions

Example 1 Determine the Laplace transform of the constant function $f(t) = 1, t \ge 0$.

<u>Calculating the Laplace Transform of Some Functions</u>

Example 2 Determine the Laplace transform of $f(t) = e^{at}$, where a is a constant.

Calculating the Laplace Transform of Some Functions

Example 3 Find $\mathcal{L}\{\sin bt\}$, where b is a nonzero constant.

Calculating the Laplace Transform of Some Functions

Example 4 Determine the Laplace transform of
$$f(t) = \begin{cases} 2, & 0 < t < 5, \\ 0, & 5 < t < 10, \\ e^{4t}, & 10 < t. \end{cases}$$

Some Properties of the Laplace Transform: LINEARITY

Linearity of the Transform

Theorem 1. Let f, f_1 , and f_2 be functions whose Laplace transforms exist for $s > \alpha$ and let c be a constant. Then, for $s > \alpha$,

$$\mathcal{L}\lbrace f_1 + f_2 \rbrace = \mathcal{L}\lbrace f_1 \rbrace + \mathcal{L}\lbrace f_2 \rbrace ,$$

$$\mathcal{L}\{cf\} = c\mathcal{L}\{f\} .$$

Some Properties of the Laplace Transform: LINEARITY

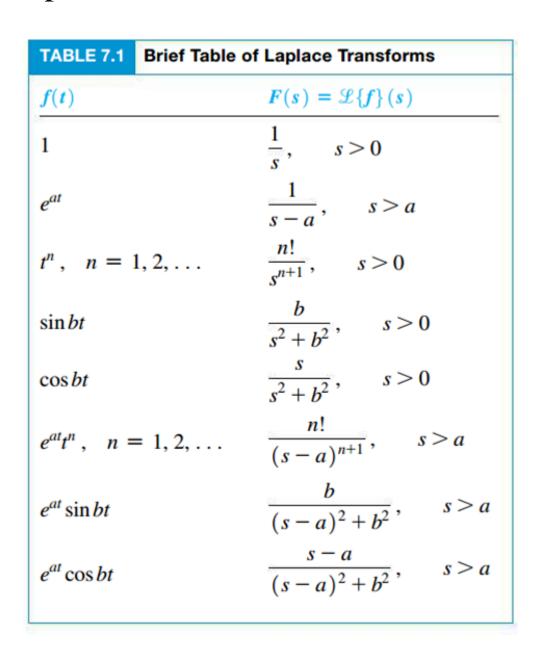
Example 5 Determine $\mathcal{L}\{11 + 5e^{4t} - 6\sin 2t\}$.

Table of Laplace Transforms

TABLE 7.1 Brief Table of Laplace Transforms	
f(t)	$F(s) = \mathcal{L}\{f\}(s)$
1	$\frac{1}{s}$, $s > 0$
e^{at}	$\frac{1}{s-a}$, $s>a$
t^n , $n=1,2,\ldots$	$\frac{n!}{s^{n+1}}$, $s>0$
sin bt	$\frac{b}{s^2+b^2}, \qquad s>0$
$\cos bt$	$\frac{s}{s^2+b^2}, \qquad s>0$
$e^{at}t^n$, $n=1,2,\ldots$	$\frac{n!}{(s-a)^{n+1}}, \qquad s > a$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \qquad s>a$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \qquad s>a$

Table of Laplace Transforms

Example 6 Use Table 7.1 to determine $\mathcal{L}\{5t^2e^{-3t} - e^{12t}\cos 8t\}$.



When Does the Laplace Transform of a Function Exist?

Piecewise Continuity

Definition 2. A function f(t) is said to be **piecewise continuous on a finite interval** [a, b] if f(t) is continuous at every point in [a, b], except possibly for a finite number of points at which f(t) has a jump discontinuity.

A function f(t) is said to be **piecewise continuous on** $[0, \infty)$ if f(t) is piecewise continuous on [0, N] for all N > 0.

When Does the Laplace Transform of a Function Exist?

Example 7 Show that
$$f(t) = \begin{cases} t, & 0 < t < 1, \\ 2, & 1 < t < 2, \\ (t-2)^2, & 2 \le t \le 3, \end{cases}$$
 is piecewise continuous on $[0, 3]$.

When Does the Laplace Transform of a Function Exist?

Exponential Order α

Definition 3. A function f(t) is said to be of **exponential order** α if there exist positive constants T and M such that

(4)
$$|f(t)| \le Me^{\alpha t}$$
, for all $t \ge T$.

When Does the Laplace Transform of a Function Exist?

Conditions for Existence of the Transform

Theorem 2. If f(t) is piecewise continuous on $[0, \infty)$ and of exponential order α , then $\mathcal{L}\{f\}(s)$ exists for $s > \alpha$.